

## PROBLEM SET 01

MARC PICKETT I

(1) (This problem was inspired by my recent experience with the Baltimore Parking Authority in the difficulty in obtaining a parking pass, which made me realize the high demand and low supply for these. That is, Federal Hill parking passes are like gold.)

In English, what is the logical consequence of the following set of statements?

- $A =$  “Marc leaves something valuable on display in his car.”
- $B =$  “Marc’s car is parked in Federal Hill.”
- $C =$  “Marc’s car gets broken into.”
- $D =$  “Parking passes are valuable.”
- $E =$  “Marc gets a \$77 parking ticket.”
- $F =$  “Marc’s car has a parking pass on display in his car.”
- $G =$  “Marc has to pay \$77 to fix his broken car window.”
- $A \wedge B \implies C$
- $B \wedge \sim F \implies E$
- $C \implies G$
- $D \wedge B$

**Answer:** From  $D \wedge B$  we get that Marc’s car is parked in Federal Hill and that parking passes are valuable. We don’t know whether Marc’s car has a parking pass on display, but either Marc leaves his parking pass on display in his car, or he doesn’t. If he does, since  $A \wedge B \implies C$ , we conclude  $C$  (since leaving a parking pass in your car is leaving something valuable in your car since parking passes are valuable) that his car gets broken into and (since  $C \implies G$ )  $G$  he has to pay \$77 to get his car

fixed. If he doesn't put a parking pass on display he gets a \$77 parking ticket (since  $B \wedge \sim F \implies E$ ). Either way, we arrive at the conclusion that **Marc has to pay \$77.**

- (2) (This problem was inspired by graffiti I read on a bathroom stall several years ago, which made an analogy between Clint Eastwood and the stall's toilet paper.)

Translate the following sentences (taken literally) into quantified logic where  $Problem(x, p, y)$  means  $x$  has problem  $p$  with  $y$ :

- (a) "Bob has problems with everybody."

**Answer:**  $\forall y \exists p Problem(Bob, p, y)$

- (b) "Bob has no problems with everybody." (In the sense that Bob doesn't have problems with anybody.)

**Answer:**  $\forall y \sim \exists p Problem(Bob, p, y)$

or

$\forall y \forall p \sim Problem(Bob, p, y)$

or

$\sim \exists y \exists p Problem(Bob, p, y)$

- (c) "Bob has problems with nobody."

**Answer:**  $\forall y \forall p \sim Problem(Bob, p, y)$

or something that's logically equivalent.

- (d) "Bob has no problems with nobody."

**Answer:**  $\forall y \exists p Problem(Bob, p, y)$

or something that's logically equivalent.

(e) “Bob doesn’t have no problems with nobody.”

**Answer:**  $\sim \forall y \exists p \text{Problem}(Bob, p, y)$

or something logically equivalent.

(3) Write the previous answer (which should have been  $\sim \forall y \exists p \text{Problem}(Bob, p, y)$ ) such that the negations appear only within predicates (i.e., no negation appears in front of a quantifier).

**Answer:**  $\exists y \forall p \sim \text{Problem}(Bob, p, y)$

(This must be exact.)

(4) BONUS: rewrite “Clint Eastwood is rough, he’s tough, and he don’t take no crud off nobody.” (taken literally) in as simplified English as possible (i.e. using at most one negative).

**Answer:** Notice that if, in question 2e, we replace “Bob” with “Clint Eastwood”, and “has problems with” with “takes crud off”, we have the phrase “Clint Eastwood doesn’t take no crud off nobody.”, which is the phrase we want to simplify (changing “doesn’t” for “don’t”).

If we define  $\text{Crud}(x, p, y)$  to mean “ $x$  takes crud  $p$  off  $y$ ”, and replace  $\text{Problem}$  with  $\text{Crud}$  and  $\text{Bob}$  with  $\text{ClintEastwood}$  in our answer to Question 3, then we get:

$\exists y \forall p \sim \text{Crud}(\text{ClintEastwood}, p, y)$

Translating this directly into English, we get: “There’s some  $y$  such that for all  $p$  Clint Eastwood does not takes Crud  $p$  off of person  $y$ .”

which isn’t quite English, so we can change it to

“There’s some person such that for all crud Clint Eastwood does not takes that crud off of that person.”

or smoothing it out some

“There’s somebody off of whom Clint Eastwood takes no crud.”

So, I’d accept answers like:

“Clint Eastwood is rough, he’s tough, and there’s someone off of whom he takes no crud.”

or

“Clint Eastwood is rough, he’s tough, and he doesn’t take crud off at least one person.”

or

“Clint Eastwood is rough, he’s tough, and there exist people he doesn’t take crud off.”

or something equivalent.

Note that, despite being a triple negative, the literal meaning is different from the stall-writer’s intended meaning, which was “There’s no one off of whom Clint Eastwood takes crud.”.