PROBLEM SET 02

YOUR NAME

(1) What is the negation of the statement "All women are strong, all men are handsome, and all children are above average."?

(2) Prove (or disprove) the following statements. *State which proof technique you used.*

   (a) "The sum of 2 consecutive integers is odd."

   (b) "The sum of 5 consecutive integers is odd."

   (c) "The sum of 5 consecutive integers is evenly divisible by 5."

(3) What can we say about sets A and B if:

   (a) $A \cup B = A$

   (b) $A - B = \emptyset$

   (c) $|A \cup B| = |A| + |B| - |A \cap B|$

(4) Translate the following Set statements into Logic. For example, the proposition $A \cup B \subseteq C$ would be $\forall x\,((x \in A \vee x \in B) \implies x \in C)$.

   (a) $(A \cap B) \subseteq E$

   (b) $(A \cap B) = \emptyset$

   (c) $(A \cap B) \subseteq (C - D)$

   (d) $(A \cap B) \cup (C - D) = E$

   (e) $A \cup B \subset C$ (Note the use of $\subset$ instead of $\subseteq$)

(5) BONUS: Prove or disprove that the product of 2 of the following numbers is non-negative: (The proof must be less than a page to receive credit.)

- $2^{2342} - 8^{780} + 3^{721}$
- $\sqrt{2}^{\sqrt{2}^{3138}} - \sqrt{3}^{\sqrt{3}^{1108}}$
- $999^{888} - 888^{999} + 777^{1020}$