PROBLEM SET 05

YOUR NAME

(1) Write a general equation for the following sequences: (For example, a general equation for the sequence $2, 5, 10, 17, 26, \cdots$ would be $a_n = n^2 + 1$. The equations should be closed form, and they should be single equations. For example "$a_n = \frac{n}{2}$ if n is even, and $a_n = \frac{n+1}{2}$ if n is odd." wouldn't be an acceptable answer for part c.)

(a) $3, 5, 7, 9, 11, \cdots$

(b) $1, 5, 7, 17, 31, 65, 127, 257, \cdots$ (Hint, remember that $(-1)^i = 1$ for even i, and $(-1)^i = -1$ for odd i.)

(c) $1, 1, 2, 2, 3, 3, 4, 4, \cdots$ (BONUS points if you can do this without using any "rounding" operators such as floor or ceiling.)

(2) Write a closed form formula for the following summations: (For example, a closed form formula for $\sum_{i=1}^{n} 2i$ would be $n\,(n+1)$.)

(a) $\sum_{i=1}^{n} (2i + 3 + 9^i)$

(b) $\sum_{i=1}^{n} \left((-1)^i \cdot 9^i\right)$

(c) $\sum_{i=1}^{n} \sum_{j=1}^{n} ij$

(d) $\sum_{i=1}^{n} \sum_{j=1}^{i} j$ (Hint, $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$. Also note that the interior summation is from $j = 1$ to i, not n.)

(3) Give a recursive definition for the following sequence: (Be sure to specify the base case(s).) $1, 1, 1, 2, 3, 4, 6, 9, 13, 19, 28, \cdots$

(4) To be a member of the Sons of the American Revolution, you have to have "at least one ancestor who supported the cause of American Independence during the years 1774-1783". Let's suppose that everyone who fits this criterion is a member.

If we define S to be the set of people who "supported the cause of American Independence during the years 1774-1783", and we define $p(x, y)$ to be "x is a parent of y", then give a recursive definition of R: the set of members of the Sons of the American Revolution in terms of S and p.

(5) BONUS: The Fibonacci numbers are defined by the equation

$$f_n = f_{n-1} + f_{n-2}$$

where $f_1 = f_2 = 1$.

Prove that, in the limit as $n \to \infty$, $\frac{f_{n+1}}{f_n} = \phi$, where ϕ is the Golden Ratio, given by $\frac{1+\sqrt{5}}{2}$.

Hint, you may want to prove that the following closed form equation gives f_n:

$$f_n = \frac{\left(1+\sqrt{5}\right)^n - \left(1-\sqrt{5}\right)^n}{2^n\sqrt{5}}$$

then use this as a *lemma* in your proof.