PROBLEM SET 06

YOUR NAME

(1) How many ways are there to get either a pair of Kings (exactly 2 kings) or a 3-of-a-kind of Aces (or both the King pair and the Ace trio) with a standard poker hand? (For this exercise, a full house with a pair of Kings counts as a pair of Kings, but 3 (or 4) Kings doesn't count as a pair of kings, and 4 Aces doesn't count as a 3-of-a-kind.)

(2) In the expansion for $(A+B)^{114}$, what constant is in front of the term $A^{62}B^{52}$?

(3) Every pack of Starburst© candy has 3 Cherry, 3 Strawberry, 3 Orange and 3 Lemon flavored Starbursts. In our class, 4 people like Cherry the best, 13 people prefer Strawberry, 8 prefer Orange, and 2 prefer Lemon. What is the minimum number of packs of Starbursts I would need to get so that I could divide the Cherry Starbursts evenly among those who prefer Cherry, the Lemon Starbursts evenly among those who prefer Lemon, etc.?

(4) If I roll 5 standard 6 sided dice, what are the odds that their sum will be 7 or fewer?

(5) The incidence of Huntington's disease (an unfortunate genetic disorder) is about 1 in 15,000. There are usually no visible signs for this disease until a person is in their late 30s or early 40s. There is a test for this disease which has a 1.8% false positive rate (i.e., $P\left(TestPositive\left(x\right) \mid \sim Huntington\left(x\right)\right) = 0.018$ where $TestPositive\left(x\right)$ means that person x gets a positive for the Huntington's test, and $Huntington\left(x\right)$ means that a person actually has Huntington's disease). If a person has Huntington's, the test result will be positive virtually 100% of the time (i.e. $P\left(TestPositive\left(x\right) \mid Huntington\left(x\right)\right) = 1.0$). Doug was adopted (and therefore we know nothing about occurrence of Huntington's disease in his biological family), and he tests positive for Huntington's.

   (a) What is the probability that he actually has this disease?

(b) BONUS: The gene that is responsible for Huntington's disease is dominant. This means that the probability of someone inheriting the disease is 50% if exactly one of their parents has the disease (and 100% in the extremely rare case where both of their parents have the disease). Suppose we find out that Doug's biological father is a carrier of this gene, but his biological mother isn't. Given that Doug's test result is positive, what is the probability that he has Huntington's disease?