

## PROBLEM SET 08

YOUR NAME

**NOTE:** This is due Wednesday, November 23rd, at 4PM. You may either turn it in in class on Tuesday, or submit it via email (marc@coral-lab.org). If you email it to me, I must receive it before 4PM Wednesday (at which point I'll be posting the answers online, and leaving to eat turkey). For the email submissions, I prefer pdf, but if you can't figure out how to make pdf, I'll accept Word docs, rtf, LaTeX source, and plain old text. (If there's another format that you plan to use that I don't have listed, let me know.)

- (1) Kompleck City is a square 9 blocks on a side. On the northeast corner is Vlad Urr's Royal Shack, and on the southwest corner is The Great Library of Kompleck City. There are 21 streets in Kompleck City. These are all one-way streets. One of the streets, called "The Zim Bob Way", goes underground from Count Urr's Royal Shack to The Great Library. The remaining 20 streets are arranged in a 10 by 10 grid. 10 of these streets (Ave.  $X_0$  through Ave.  $X_9$ ) go from south to north, and 10 (Ave.  $Y_0$  through Ave.  $Y_9$ ) go from west to east. For every day of his reign, Count Urr goes to the Royal Library (along The Zim Bob Way), then returns along a different path to survey his realm.

(An example path would be where Count Urr takes Zim Bob Way to The Great Library (at the intersection of  $X_0$  and  $Y_0$ ), then Ave.  $X_0$  to Ave.  $Y_1$ , then  $Y_1$  all the way to  $X_3$ , then  $X_3$  to  $Y_9$ , then  $Y_9$  back to his Shack at the intersection of  $X_9$  and  $Y_9$ .)

- (a) If Count Urr decides to represent a map of Kompleck City as a graph, how many nodes and how many edges should that graph have? (Hint, were Kompleck City

1 block on a side, 2 by 2, or 3 by 3, the number of edges and nodes would be 5 and 4, 13 and 9, and 25 and 16, respectively.)

- (b) How long must Count Urr’s reign last if he is to take every possible path through his realm? (Hint, you might want to try this on a smaller grid first. For example, the number of paths, were Kompleck City 1 by 1, 2 by 2, and 3 by 3, would be 2, 6, and 20, respectively.)

(2) A *undirected hyperedge* is like an ordinary undirected edge except it may be between<sup>1</sup> any number of nodes (well, any number  $\geq 2$ ). *Undirected hypergraphs* are a superset of regular undirected graphs that may contain hyper-edges (so all undirected graphs are hypergraphs). For example, I can have a graph that has 4 nodes:  $A$ ,  $B$ ,  $C$ , and  $D$ , and the following hyperedges:

- $\{A, B\}$  (This is just a regular undirected edge between  $A$  and  $B$ )
- $\{C, D\}$  (Another regular edge between  $C$  and  $D$ )
- $\{A, C, D\}$  (A hyperedge among (“bethreen”?)  $A$ ,  $C$ , and  $D$ . Note that, since this is undirected, this is the same as the hyperedge  $\{C, D, A\}$ .)
- $\{A, B, C, D\}$  (A hyperedge befourn  $A$ ,  $B$ ,  $C$ , and  $D$ .)

(a) How many plain<sup>2</sup> undirected hypergraphs are there that have:

- (i) 2 nodes: ( $A$  and  $B$ )
- (ii) 3 nodes: ( $A$ ,  $B$ , and  $C$ )
- (iii) 4 nodes: ( $A$ ,  $B$ ,  $C$ , and  $D$ )

(b) Write a general expression for the number of plain undirected hypergraphs that have  $n$  nodes. (Hint, you may use the  $\sum$  operator, but not ellipses “ $\dots$ ”.)

(c) BONUS: Write a *closed form* general expression for the number of plain undirected hypergraphs that have  $n$  nodes. (You can use this answer here for part b too.)

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<sup>1</sup>Grammatically and etymologically, I should write “among”, since between is specific to 2 things.

<sup>2</sup>By “plain” I mean without self loops or multiple edges. The hyperedges  $\{A, B, C\}$  and  $\{A, B\}$  wouldn’t count as multiple edges.