

PROBLEM SET 09

MARC PICKETT I

(1) Using Huffman's algorithm, how many bits would it take to encode the string:

AN_ANT_NEST_TENANT_SET_TEN_ANTENNA_TENTS

(Not including the number of bits to encode the encoding tree itself.) Note that “_” is also a character.

Answer: We note that there are the following frequencies in the string:

10 N's

9 T's

7 _'s

6 E's

5 A's

3 S's

Given this frequency distribution, the encoding tree should have the following structure (though 0's might be where I used 1's and vice versa):

N = 00

T = 10

_ = 010

E = 011

A = 110

S = 111

If we use this and make the replacements, we get

11000010110001001000011111100101001100110001001011

101110010100110001011000100110000110010100110010111

which is **101** bits long.

(2) Write a logical expression that's equivalent to the following but uses only $\bar{\wedge}$ s (NAND).

(a) $\sim A$ **Answer:** $\sim A = A\bar{\wedge}A$

(b) $A \wedge B$ **Answer:** $A \wedge B = (A\bar{\wedge}B)\bar{\wedge}(A\bar{\wedge}B)$

(c) $A \vee B$ **Answer:** $A \vee B = (A\bar{\wedge}A)\bar{\wedge}(B\bar{\wedge}B)$

(3) A universal Boolean operator is one (such as NAND) that can be used to define any other Boolean operator. Prove or disprove the following (you can assume that NAND is universal, since we just showed that you can use it to make AND, OR, and NOT):

(a) NOR ($\bar{\vee}$) is universal (hint, the computer that was used to guide the Apollo space mission was built using only NOR gates).

Answer: This is true. Proof: we can make AND from NOR gates:

$$A\bar{\wedge}B = ((A\bar{\vee}A)\bar{\vee}(B\bar{\vee}B))\bar{\vee}((A\bar{\vee}A)\bar{\vee}(B\bar{\vee}B))$$

We can now make any other operator by substituting the above expression wherever we have NAND.

(b) (BONUS) XOR is universal.

Answer: This is false because it's impossible to make NOT using only XOR. To see this, suppose A is FALSE, and we want to make $\sim A$, which should be TRUE. The only operation we could apply to A would be $A \text{ XOR } A$ which would be FALSE as well. Thus, there would be no way to get a TRUE signal.

(4) (This is exercise 11.14) A palindrome is a string that reads the same backward as it does forward. Find a context-free grammar that generates the set of all palindromes over the alphabet $\{A, D\}$.

Answer:

$\langle S \rangle ::= A \langle S \rangle A \mid D \langle S \rangle D \mid A \mid D \mid (\text{NULL})$

- (5) On a scale from 0 to 1, what do you think your attendance/participation grade ought to be? Please briefly justify your answer.

Answer: I think I should get a .99 because I showed up on time to almost every class, and I tried to be an active participant in the class discussions.

Participation/attendance, as 10% of the grade is worth 250 points. I graded this on 2 criteria: participation and attendance (as you may have guessed).

Coincidentally, everyone who turned in solutions to problem set 9 (this one) got at least $\frac{250}{250}$. There were some people who had high attendance, but who weren't very vocal in class, and some people who missed several classes, but who had heavy participation when they came to class. These people got full points. There were also people who had both high attendance and who participated heavily, and these people earned up to 50 bonus points ($\frac{300}{250}$).